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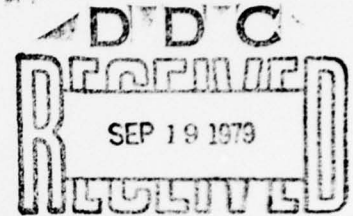QUALITATIVE STUDY OF NONLINEAR PROBLEMS IN ANALYSIS

Final Technical Report No. 1

1 Jan 66 - 30 Jun 79

by Jane Cronin Scanlon

August 30, 1979

30 Aug 79

U. S. Army Research Office

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The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

fixed points, topological degree, nonlinear problems, abstract functional equations, recurrent and periodic solutions of nonlinear ordinary differential equations, stability, mathematical physiology, periodic catatonic schizophrenia, aneurysm of the circle of Willis.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The objective of the work was to use qualitative methods, especially fixed point theorems and topological degree, to study nonlinear problems in analysis. Existence of solutions of wide classes of abstract nonlinear functional equations are established and the existence of recurrent and period solutions of nonlinear ordinary differential equations are obtained. Upper bounds for the

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20. Abstract (Continued):

number of such solutions are also obtained. (Such upper bounds for the number of periodic solutions of nonlinear ordinary differential equations have become significant in mathematical studies of chemical reactions.) Several results in the stability of periodic solutions of nonlinear ordinary differential equations were also established.

Several mathematical problems which arise in physiology were studied. The most specific conclusion obtained is a clinical recommendation for the treatment of patients suffering from a rupture of an aneurysm of the circle of Willis. Finally, two books have been written: a textbook on qualitative theory of ordinary differential equations and a set of lecture notes on the mathematics of cell electrophysiology.

The objective of the work was to use qualitative methods (especially fixed point theorems and topological degree) to study nonlinear problems in analysis. The problems studied range from existence of solutions of abstract functional equations to the number of periodic solutions of specific nonlinear ordinary differential equations which arose from studies in physiology.

Existence of solutions is studied in a number of papers [1, 3, 7, 10, 14, 15, 16, 20, 25, 26, 28]. In [1, 3, 7, 9, 14, 15, 16, 20, 25, 28], recurrent and periodic solutions of ordinary differential equations are studied. In [10, 15, 16, 26], solutions of other concrete functional equations and abstract functional equations are studied.

In the papers [4, 5, 6, 11, 12, 21, 23], topological degree is used to obtain an upper bound for the number of distinct solutions of nonlinear equations or to estimate the number of distinct solutions. In [4, 5, 6, 12], abstract functional equations are studied. In [11, 21, 23] periodic solutions of ordinary differential equations are studied. (It should be noted that obtaining upper bounds for the number of solutions has become a problem of interest in mathematical studies of chemical reactions.)

Papers [8] and [31] are concerned with stability studies. Paper [31] presents a computable criterion for the existence of a phase asymptotically stable solution of a nonlinear system of ordinary differential equations. This result is of interest because when combined with an earlier result in [20] it yields the existence of a phase asymptotically stable periodic solution.

Papers [13, 17, 18, 19, 24, 29, 30] are concerned with analysis of specific nonlinear differential equations which arise in studies in physiology. Paper [19] presents the most specific conclusion. A fairly abstract analysis, which uses topological degree, yields a specific clinical recommendation for treatment of patients suffering from a rupture of an aneurysm of the circle of Willis. The clinical recommendation is new and was of interest to Dr. Paul Slosberg, an internationally known medical specialist in this field.

Item [2] concerns a topological problem. Items [22] and [27] are expository accounts.

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<u>1/1/66</u>   ---   <u>6/30/69</u>

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* Also supported under previous Army grant; see page 1

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